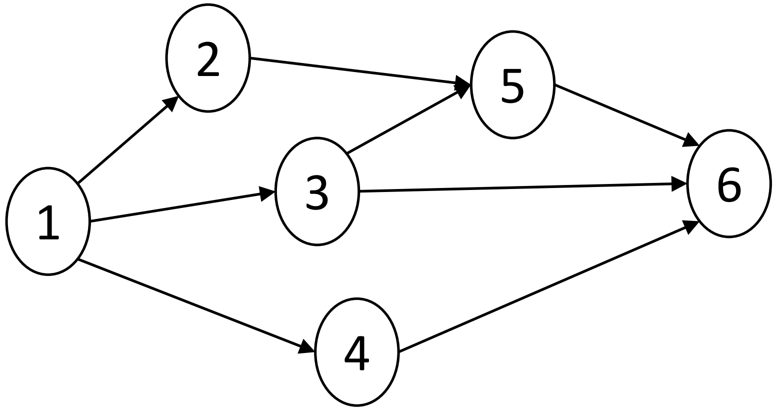
# A Divide and Conquer Algorithm for DAG Scheduling under Power Constraints


Gökalp Demirci Ivana Marincic Henry Hoffmann

THE UNIVERSITY OF CHICAGO

Department of Computer Science

# Scheduling DAGs under Power Constraints




Scheduler


# Scheduling DAGs under Power Constraints

Consider n tasks

Both constraints are easy to handle separately (to get theoretical bounds):

**DAG** scheduling:
- Precedence constraints
- Any greedy algorithm: 2-approx [‘66]
- Given >50 years ago

**Power** constrained scheduling
- Independent tasks
- Any greedy algorithm: 3-approx [‘75]
- Similar to packing problems

+

Scheduling **DAGs** under **Power** Constraints:
- No formal bounds until very recently
- Greedy algorithms could be O(n)-approx
- O(log n)-approx using a divide-and-conquer technique [Demirci, Hoffmann, Kim’18]

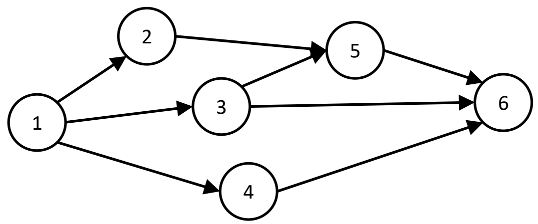
This paper:

Scheduling **DAGs** under **Power** Constraints on **Configurable** Hardware (e.g., DVFS)

# Scheduling DAGs under Power Constraints

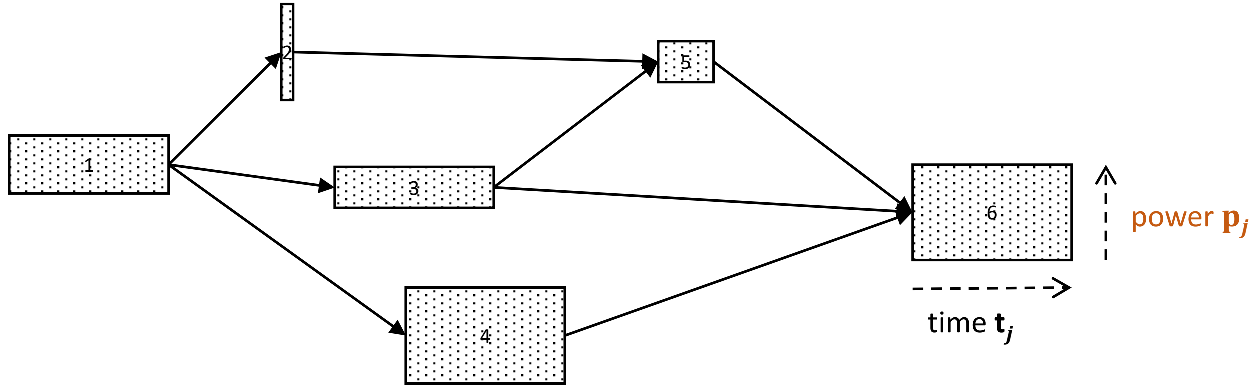
Schedule $n$ tasks on $m$ parallel machines to minimize total run-time

**DAG** constraints
- If $j \prec j'$: finish $j$ before starting $j'$
- Task based programming, workflow management, precedence constraints…

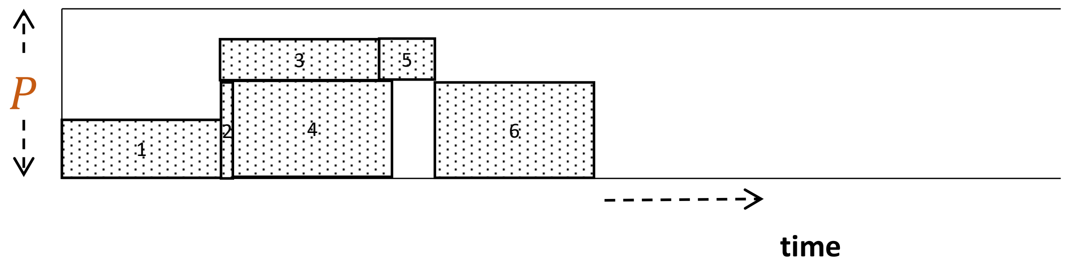



\+

**Power** constraints
- Each task $j$ needs $\mathrm{p}_j$ amount power
- Total available power at any point: $P$
- Tasks running in parallel $\sum_j \mathrm{p}_j \leq P$

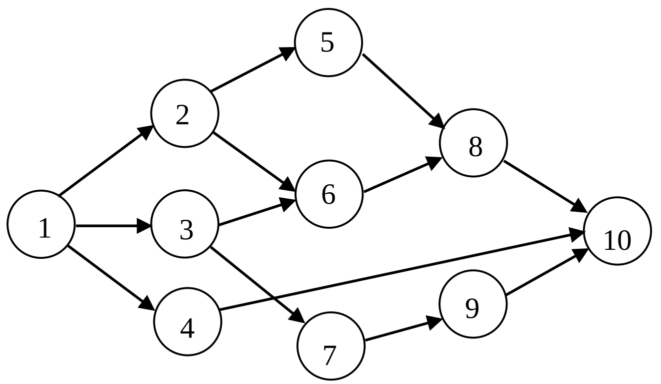


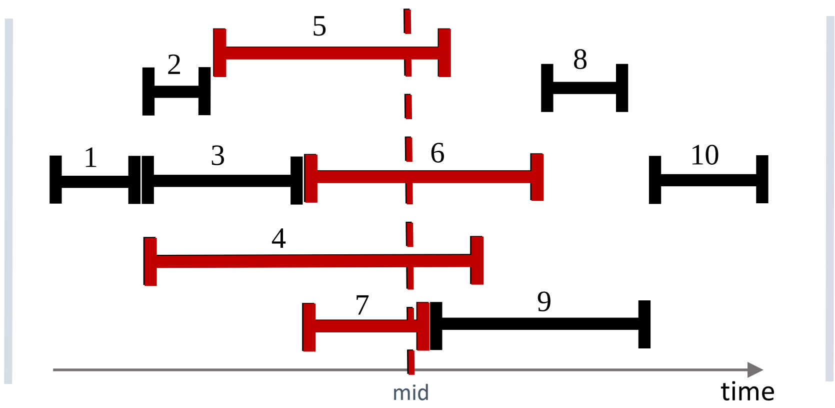

Start with the simpler problem:
No configurations – e.g. fixed ($\mathbf{t}_j$, $\mathrm{p}_j$)

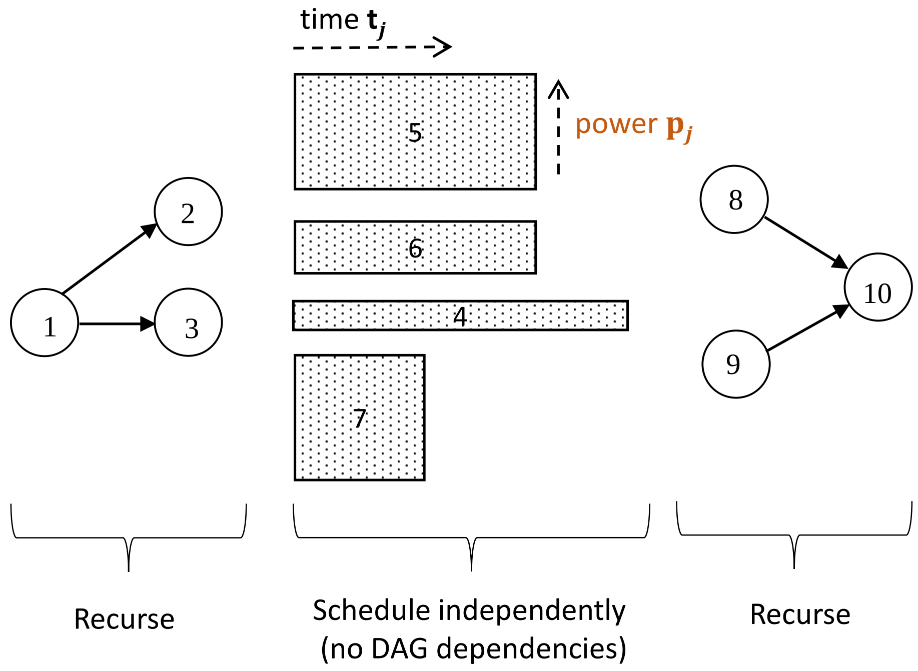


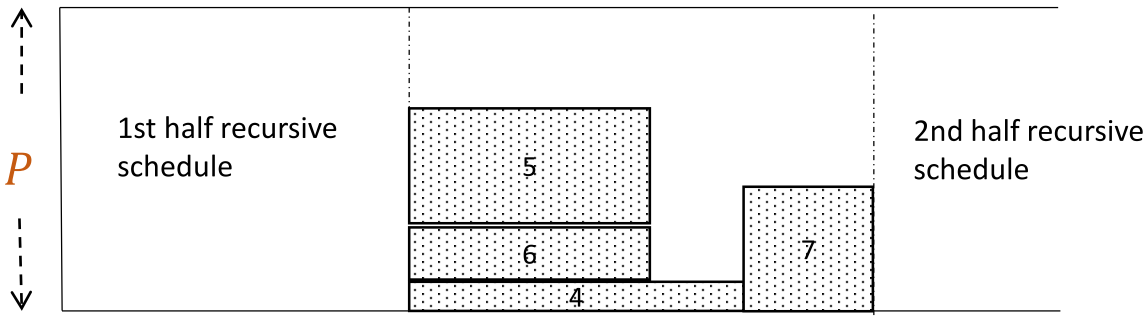
# Scheduling DAGs under Power Constraints

## D&C Algorithm: No configurations, e.g. fixed ($\mathbf{t}_j$, $p_j$)



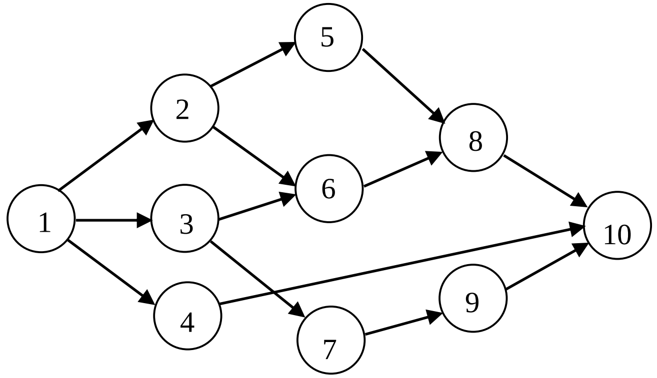



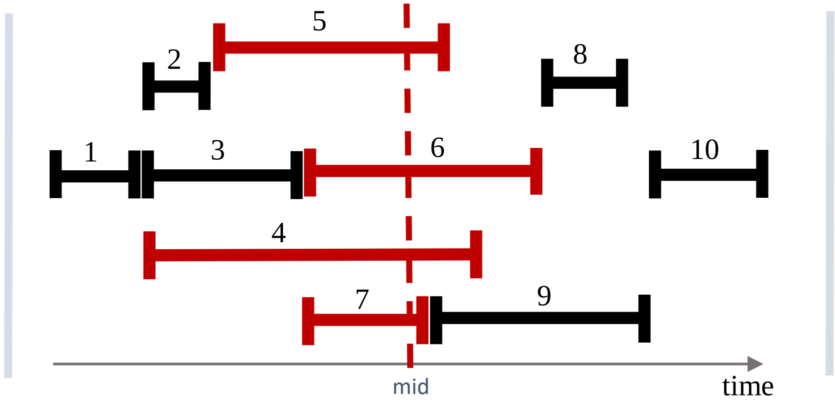



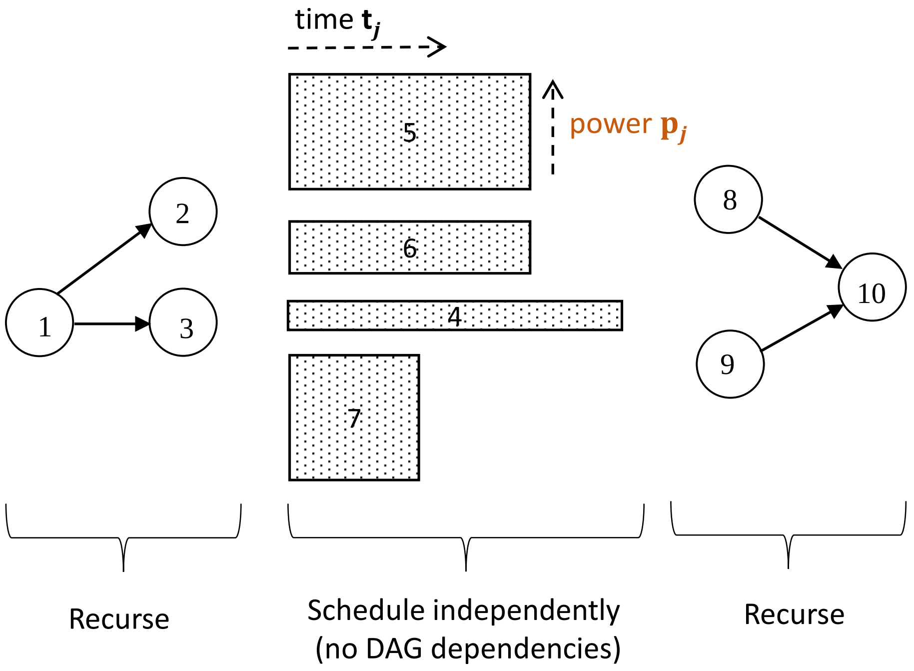

- Get a schedule satisfying the DAG w/ ∞ parallelism ignoring the power constraints
- Consider the subset of tasks crossing the mid point: no DAG dependencies among them
- Schedule them in a separate fragment in the middle (greedily) now considering power
- Recurse on both sides

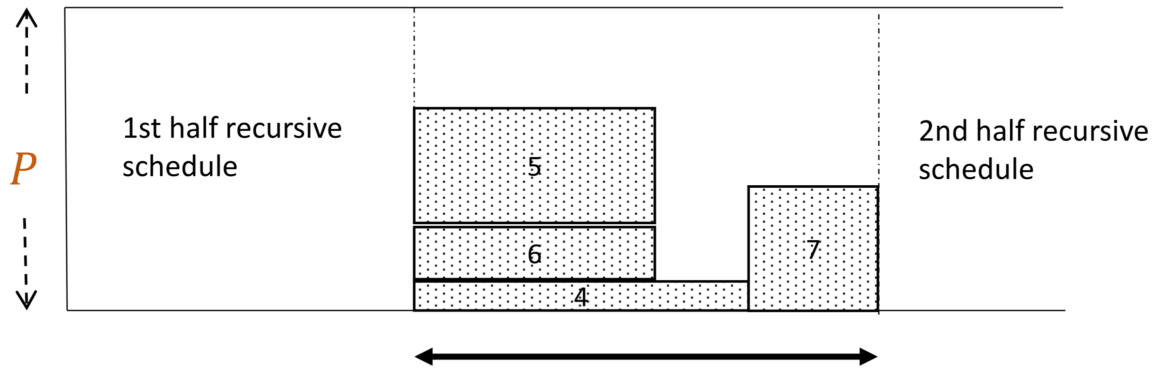
# Scheduling DAGs under Power Constraints

## D&C Algorithm: No configurations, e.g. fixed ($\mathbf{t}_j$ , $p_j$)




- O(1) loss in every level of the recursion
- Depth of the recursion: O(log n)
- ✓ O(log n) - approximation

Analysis idea:

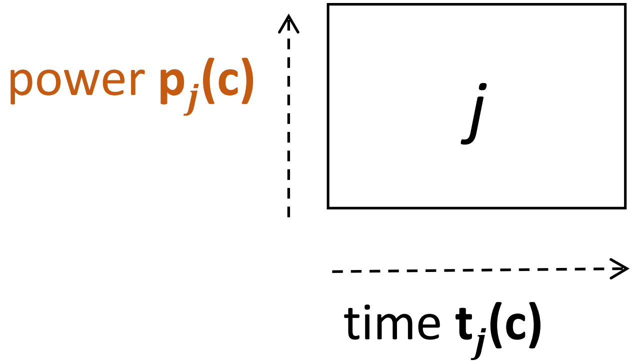
Length of the fragment <≈ max-time + total area/$P$

<≈ O(1) × optimum schedule length

# Scheduling DAGs under Power Constraints

## Scheduling with ***Configurations:***

- Each task can be one of many rectangles (determined by configurations)
  - Length is time
  - Height is power



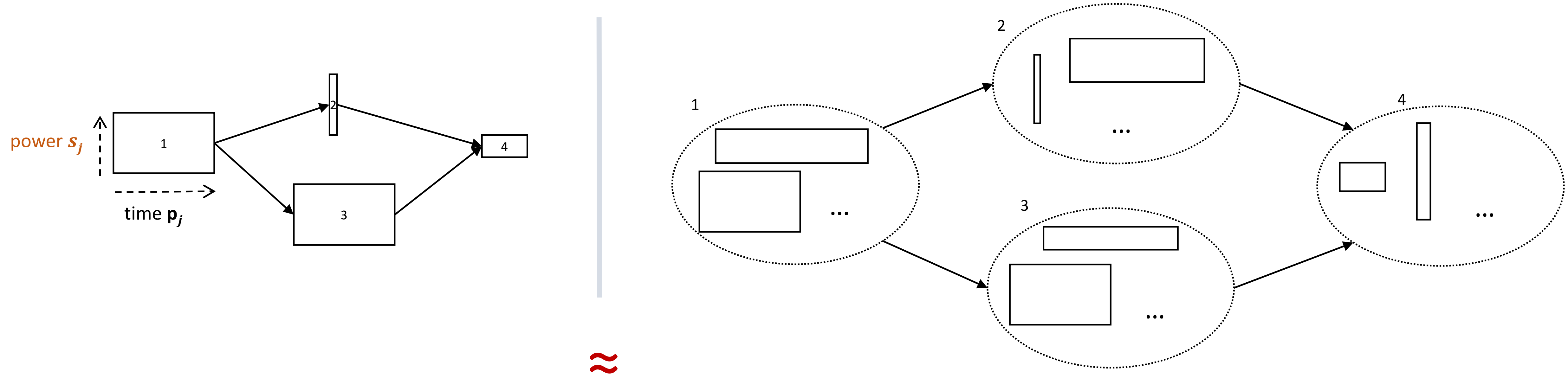

- Intuition:
  - Tallest rectangle completes work fastest (*race* configuration)
  - Smallest area rectangle is most energy efficient (*pace* configuration)
  - Greedy schedulers have to favor one or the other

✓ We can choose **configurations** such that it schedules within O(log n) of optimum!

# Scheduling DAGs under Power Constraints

## Scheduling with ***Configurations:***

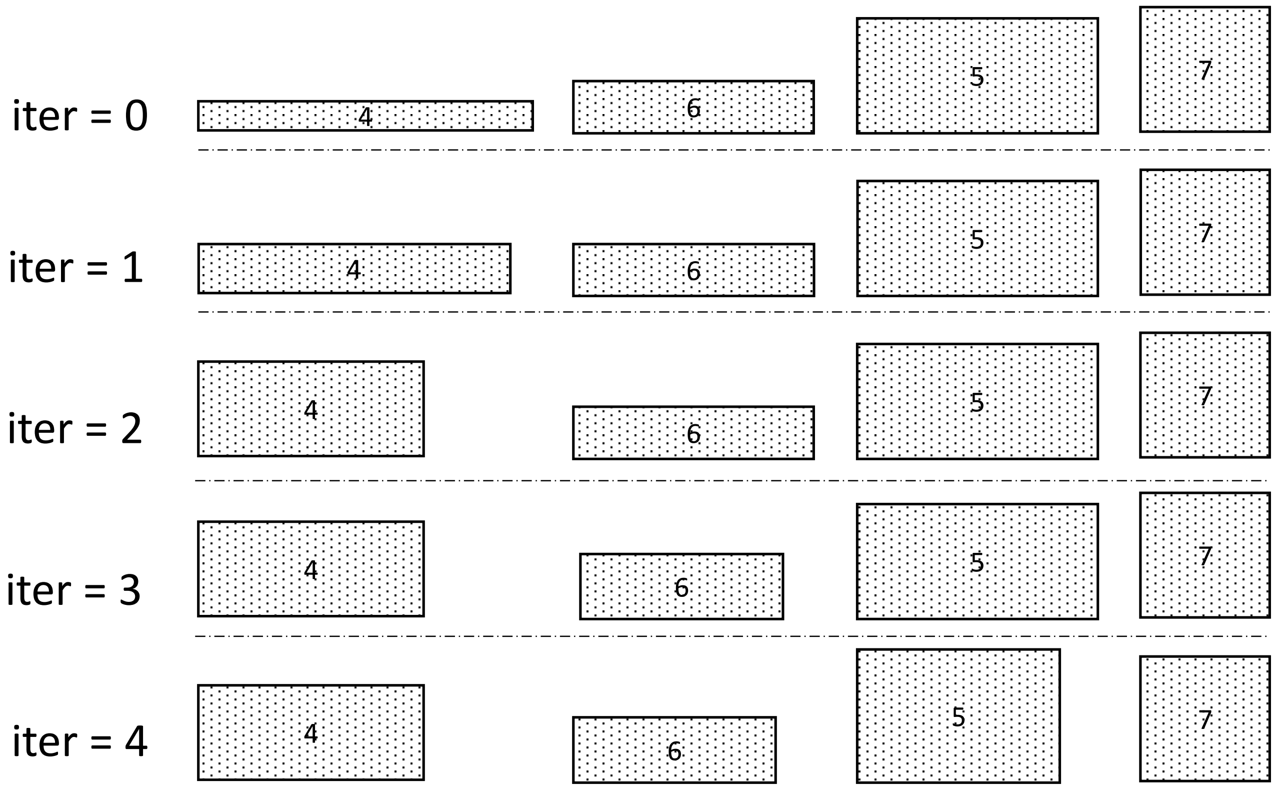
- Motivated by HPC systems: hardware configs with different power/run-time tradeoffs
  - (2 active cores, high DVFS, 2 mem. contr.) vs (4 active cores, low DVFS, 1 mem. contr.)

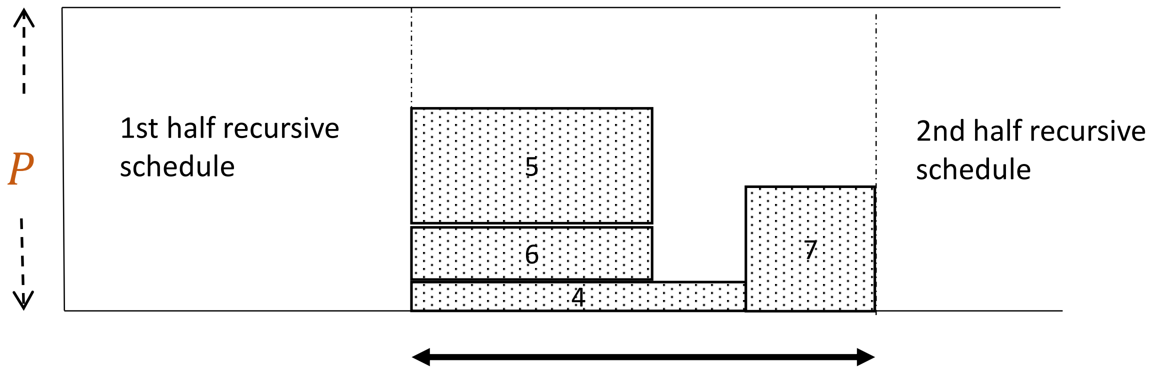



✓ We can choose **configurations** such that it schedules within O(log n) of optimum!

# Scheduling DAGs under Power Constraints

## Scheduling with ***configurations:*** (≈ no configurations)




$<\approx \max\{\mathbf{t}_j\} + \text{total area}/P$

- Not hard to argue there exists an iteration where
  1) max-time ≤ max-time$_{OPT}$ (Tall rectangles)
  2) area-of-each ≤ area-of-each$_{OPT}$ (Minimal area rectangles)

- Our configs need to be comparable to opt. schedule`s configs in terms of 1) max-time and 2) total area
- Start with pace configurations (min area configurations)
- Enumerate set of configs by replacing only the max-time task in every iteration
- Replace it w/ the configuration that has the smallest area among configs “shorter” than the current

# Scheduling DAGs under Power Constraints

Greedy Algorithms:

- Only major design choice is the order in which available tasks are considered
- Three popular choices in state-of-the-art **DAG** schedulers or **Power** schedulers:
  - G1: first in (the queue of available), first out
  - G2: best-fit to power budget, i.e. pace power closest to available power
  - G3: non-increasing run-time (in pace configuration) order
- We implement and compare our D&C algorithm to all three

# Scheduling DAGs under Power Constraints

Percent improvement of D&C over **best of** G1, G2, G3 in each setting:

Different DAGs →

*#machines=10*

| | backprop | kmeans | npb-dc | npb-is | swift1 | swift2 | synth-lg-long |
|---|---|---|---|---|---|---|---|
| *P=100W* | 60.3% | 60.0 | 67.4 | 64.6 | 63.9 | 72.0 | 32.7 |
| *P=200* | 35.4 | 36.9 | 36.2 | 43.8 | 40.4 | 46.2 | 13.4 |
| *P=300* | 5.8 | 18.5 | 9.9 | 15.5 | 4.5 | 3.6 | 5.0 |
| *P=400* | 3.8 | 3.1 | -7.2 | 11.8 | -12.2 | 0.0 | 2.3 |
| *P=500* | -8.2 | 5.3 | -15.7 | -3.0 | -14.3 | 2.4 | -18.8 |

*#machines=20*

| | backprop | kmeans | npb-dc | npb-is | swift1 | swift2 |
|---|---|---|---|---|---|---|
| *P=200W* | 62.3% | 61.5 | 59.8 | 72.8 | 74.6 | 64.3 |
| *P=400* | 39.4 | 37.8 | 49.0 | 39.2 | 27.7 | 30.0 |
| *P=600* | 2.3 | 12.9 | -8.7 | 8.2 | 10.0 | -13.2 |
| *P=800* | 1.9 | 3.4 | -27.2 | 5.6 | 6.8 | -20.4 |
| *P=1000* | -25.5 | -10 | -15.5 | -9.3 | -2.3 | -24.7 |

mean, median pace power ≈ 20W

We have more results in the paper!

As we relax the power-cap,
- Problem turns into no power DAG scheduling
- D&C algorithm (designed for power-cap) loses its edge

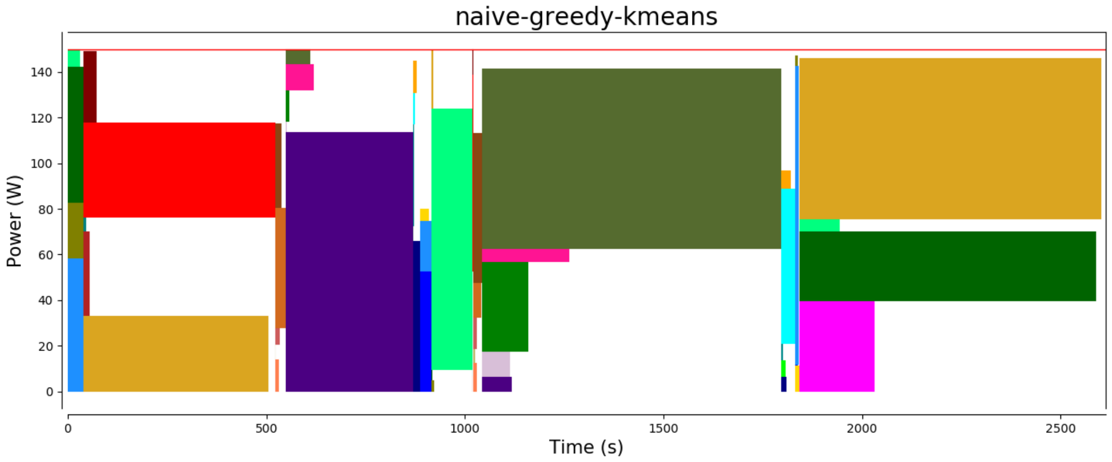
# Scheduling DAGs under Power Constraints

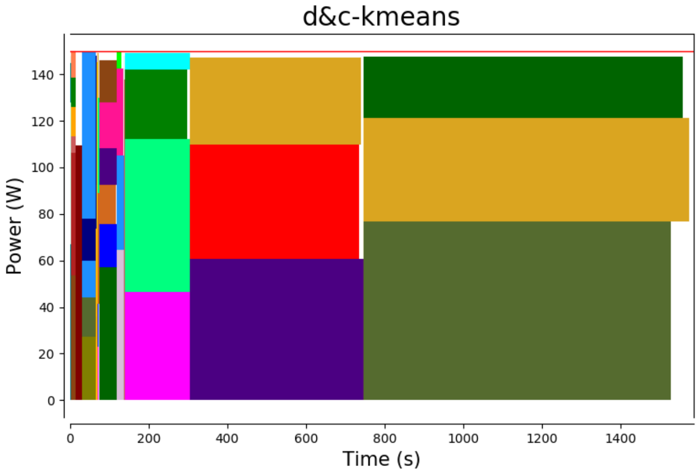
Edge of D&C explained:

To achieve min total run-time: (for any algorithm)

1) High power budget utilization (close to race **configs**), and
2) Low energy consumption of individual configurations (close to pace **configs**)

D&C aligns tasks with “similar run-times” together which, in turn, allows it to achieve high power utilization (1) without having to deviate from low energy configs too much (2)     -----------  (1) and (2) **simultaneously**!



## Takeaways:

- “Scheduling **DAGs** under **power** constraints” has a significantly different structure than just “scheduling **DAGs**”

- Greedy algorithms have no formal bounds

- D&C algorithm:
  - ✓ has a formal bound
  - ✓ performs better than greedy, because
    - ✓ high power utilization without having to deviate from pace configs too much
  - ✓ aligns similar run-time tasks in the same batch and runs batches w/o interleaving
    - ✓ promising for other future applications to **DAG** scheduling possibly w/o resources